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An Application of Bayes' Theorem to a Problem of Cultural Astronomy Interest

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Abstract

In this paper, an elliptical enclosure, found at Piani d'Avaro (Bergamo Province, Lombardy, Northern Italy) was examined from an astronomical point of view. Due to earlier studies by one of the authors (Gaspani), its internal structures were known to be astronomically aligned - and suspected to date back to the Iron Age - but the external enclosure was thought to be Late-Medieval. Using a dating algorithm, devised by the authors, it was confirmed that the site is astronomically aligned. Lunar and solar alignments are reported, among which we mention those at sunrise and sunset on the four main Celtic festivals. We found out that the site had two building phases. In the first phase, 510±20 BC, the internal structures were erected and aligned. In the second phase, 340±20 BC, the elliptical enclosure was built and aligned. An innovative application of Bayes' theorem allowed us to estimate the probability that the enclosure was dimensioned by a priestess; this probability was found to be (55±8)%. This suggests that, with a probability of (72±8)%, there were both male and female druids (in the same sense as in the rest of Celtic Europe?) in Cisalpine Gaul. More significantly, we found out that the Bayes factor, estimated to be greater than ~(5.7±0.8) according to the Kass-Raftery scale, suggests that the structures provide "more than substantial" evidence for the presence of druidesses in Cisalpine Gaul.

Keywords: Bayes' theorem, Celts, Cisalpine Gaul, Dating, Druids, Enclosure, Sanctuary.

Introduction

The Monte Avaro Barec (Fig. 1) is an elliptical enclosure located in Val Brembana (Bergamo Province, Lombardy, Northern Italy). Thanks to previous studies by one of the authors (Gaspani 2001), its internal structures were known to be astronomically aligned and probably date back to the Iron Age; the enclosure, however, was thought to date back to the Late Middle Ages. The structure was remeasured and analysed by the authors and the results went beyond their expectations; in fact, it was possible to date the site but also read the different phases of its construction and obtain indications on the gender of its designer (Gaspani and Spagocci 2018, 2020, 2022). In this paper, we give an account of these studies, in particular dealing quantitatively with Bayes' theorem. In the following, errors are expressed as 95% confidence limits.

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Site alignments

We found both lunar and stellar alignments. Lunar alignments were checked using the SkyMap simulation program. As for stellar alignments, we checked 144 targets – the stars above the third magnitude visible from our latitude - using the Rigel sky simulator, written in Fortran by one of the authors (Gaspani). The structure was measured by establishing a GPS base and referring to it the measurements made with a compass. *Google Earth* was also employed for some measurements. For a description of the procedure, see (Cernuti and Gaspani 2006). Figure 2 shows the alignments found (Gaspani and Spagocci 2018, 2020).

The coordinates of the centre of the ellipse (marked on the terrain by a stone which seems to be buried for most of its height) that approximates the profile of the enclosure, measured with *Google Earth*, are:

Latitude:	46° 00′ 37.90″ N
Longitude:	9° 35′ 51.23″ E
Altitude (ellipsoid WGS84):	1761 m

The profile of the structure was approximated by an ellipse, described by the following set of parameters:

Major semi-axis:	32.30 m
Minor semi-axis:	23.42 m
Eccentricity:	0.69

The azimuths of the axes of the ellipse were measured with *Google Earth*, with the following results (here and below the horizon height, determined with the help of the *Peakfinder* software, was considered):

Semi-major axis (AC):	(159.0 ± 0.4)°	horizon height = -1′
Semi-major axis (CA):	(339.0 ± 0.4)°	horizon height = +22′
Semi-major axis (BD):	(249.0 ± 0.6)°	horizon height = +2′
Semi-major axis (DB):	(69.0 ± 0.6)°	horizon height = +3′

The azimuth of the astronomically significant lines, formed by the internal monoliths, was instead measured on the field by a team led by Gaspani, with the result:

EF Line:	(141.7 ± 0.5)°	horizon height = 0′
FG Line:	(220.5 ± 0.8)°	horizon height = +2′

The axes of the ellipse (determined by fitting the enclosure profile to an ellipse) were oriented towards the following targets (the dates are given according to the Julian calendar, projected backwards):

AC: Heliacal rising of Fomalhaut (April 20)

BD: First morning set of Rigel (October 30)

The following solar alignments were also found at dates compatible with the main Celtic festivals:

DB: Sunrise at Beltane and Lughnasad

BD: Sunset at Imbolc and Samhain

As for the Celtic festivals, a few remarks are in order (Gaspani and Spagocci 2020). That the Celtic festivals are not only Irish is shown (at least for Samhain) by the well-known Coligny calendar (Gaspani 2012, p. 92-93). On the other hand, Gaspani and Cernuti (Gaspani and Cernuti 1997) analysed Iron Age alignments all over Celtic Europe (including Northern Italy) and, on this basis, found a few star alignments that seem to have been important for the ancient Celts. Among them there are the heliacal risings of Antares, Capella, Aldebaran and Sirius; the dates and time spacing of these events led Cernuti and Gaspani (Cernuti and Gaspani 2006, p. 26-32) to the conclusion that they respectively marked the four Celtic festivals.

Alignments to the extreme positions of the Moon were also found:

EF: Moon rising at $\delta=(-\varepsilon-i)$

FG: Moon setting at $\delta=(-\varepsilon-i)$

The dating algorithm

With the aim of dating this and other structures, we devised an algorithm based on astronomical alignments (Gaspani and Spagocci 2022). In our algorithm, we define a time interval, centred on the real time of the alignment, in which an ancient astronomer could have identified the alignments. Ancient observers, in fact, due to the limited precision of the measurements, would not have been able to distinguish the real position of a star from those having an azimuth located within a certain tolerance band. To take this into account, an azimuth tolerance is defined; the alignments, therefore, are verified in a time interval centred on the year in which the extreme position occurs and whose width is given by a few years (echoes), the number of which must be determined by simulating the sky at the relevant times.

The algorithm is based on the observation that in the case of a non-astronomically oriented artefact, there is a certain probability that one or more of the architectural alignments point to one of the possible astronomical targets. We thus have a statistical background of pseudo-events, in the sense that for each time interval considered there will be a certain number of apparent alignments that will fluctuate from interval to interval, according to a Gaussian distribution whose mean value and standard deviation was calculated in (Gaspani and Spagocci 2022). The true alignments will then emerge from the statistical background, giving rise to peaks whose profile has been determined to be approximately Gaussian.

The dating of the artefact corresponds to the time at which the Gaussian has a peak, and the width of Gaussian gives the dating error. Based on the Poisson distribution, it is then possible to calculate the probability that a random fluctuation simulates a peak amplitude equal to the one we observed. Those peaks that give a statistical fluctuation probability of less than 5% are considered reliable. By graphing the statistical fluctuation probability versus time, a plot equivalent to that obtained by graphing the number of alignments versus time is obtained, except that the dating of the site is indicated by an inverted peak.

A sensible strategy for lunar alignments turned out to be to look at the five years before and after the year in which the true extreme position occurs. Due to the limited number of targets –

eight, the extreme positions of the Moon – it is possible to manually check the alignments, using a simulation program such as *SkyMap*.

Star alignments, in principle, could be treated in the same way; however, the sample size for some target stars is too small to be employed in a statistical analysis. A different approach was followed to overcome this difficulty; the *Rigel* simulator (written in *Fortran* by Gaspani) was in fact used to list the alignments with the 144 stars visible from our latitude, whose visual magnitude is greater than the third. *Rigel*, based on a neuro-fuzzy system, provides the list of specified target alignments with the 144 most visible stars; the neural network energy is also calculated, the energy being defined as the sum of the squares of the alignment errors between each architectural line and each target star.

Structure dating

Figure 3 and Figure 4 show the dating curves obtained respectively for the elliptical enclosure and its internal structures (Gaspani and Spagocci 2018, 2020, 2022). The dating curves were obtained by applying our algorithm with a tolerance on alignments of ±0.5° (this value is justified by considering two poles with a diameter of 5 cm and placed 10 m apart, in any case it was found that the dating is not sensitive to the precise value of the tolerance).

The dating curve of the internal structure for lunar alignments (Fig. 3) shows two peaks at 510±20 BC and 340±20 BC, with a randomness probability of 0.03%. The dating curve of the outer structure for stellar alignments (Fig. 4) shows two peaks at 600±100 BC and 320±140 BC, with a randomness probability of 0.7%. The dates calculated for internal and external structures are statistically compatible; we then averaged them.

Based on these considerations, we state that the site was astronomically aligned and had two building phases. In the first phase, 510±20 BC, the internal structure was erected and aligned. In the second phase, 340±20 BC, the outer enclosure was erected and aligned. The randomness probability for the alignments turns out to be 0.7%. The proposed dates seem reasonable, as the randomness probability turns out to be much smaller than the maximum allowable level, fixed, as customary in statistics, at 5% (Gaspani and Spagocci 2020, 2022) and the results for lunar and solar dating are compatible. This dating exercise, therefore, allowed us to create an “astronomical stratigraphy” for the site (Gaspani and Spagocci 2018, 2020, 2022).

The optimization algorithm

We also devised an algorithm (Gaspani and Spagocci 2018, 2020) to estimate the unit of measurement used for sizing the structure. This was done by considering that the a/b ratio of the ellipse with which the profile of the enclosure is approximated (Fig.2) is close to 4/3, so the Pythagorean triplet (3,4,5) was probably employed.

Adding the measurements of the semi-major axis of the ellipse, its semi-minor axis and the diagonal of the relative Pythagorean triangle and dividing the result by 12, we obtain a multiple of the unit of measurement probably used to size the structure. The result, in our case, was 7.97 m. Once the multiple of the unit of measurement is calculated, one must choose between its possible submultiples. In this regard, the algorithm finds the submultiple which, multiplied by a whole number, more accurately approximates the perimeter of the Pythagorean triangle.

The optimization resulted in a unit of measurement equal to 0.50 m; this is a reasonable value, provided that the builders materialized the Pythagorean triangle by means of a rope with a few segments (delimited by knots) multiple of 12, arranged in a right triangle whose sides have measurements multiples of (3, 4, 5).

We also considered the possibility that, instead of a rope with knots, human steps were used to size the Pythagorean triangle. There is a relationship between the stride length and the height of a human being, as the height is equal, within ~10%, to twice the stride length (Maddalena 2010); stride lengths compatible with the height of a human being were calculated to be 0.89 m and 0.80 m. The height of the druid could then be either 1.78 m (a man?) or 1.60 m (a woman?).

A primer on Bayes' theorem

Since the reader is not necessarily familiar with Bayes' theorem, we give an elementary primer on Bayesian statistics, based on (Stone 2013). A typical problem suitable to be attacked by Bayes' theorem is the following: we have "n" alternative theories T_j, with a-priori probabilities $P(T_j)$, which must be checked against a piece of evidence E, with a-priori probability P(E), with the aim of calculating the conditional probability that the theory T_i is valid given the piece of evidence E, $P(T_i|E)$.

The conditional probabilities are thus defined:

$$P(T_i|\,E)P(E) = P(T_i \cap E) \tag{1}$$

$$P(E|\,T_i)P(T_i) = P(E \cap T_i) \tag{2}$$

From the fact that $P(T_i \cap E) = P(E \cap T_i)$, and Eq. (1) and (2), one then finds the well-known expression of Bayes' theorem:

$$P(T_i|\,E) = \frac{P(E|T_i)\,P(T_i)}{P(E)} \tag{3}$$

where E is a normalization constant, given by:

$$P(E) = \sum_{j=1}^{n} P(E \mid T_j)\,P(T_j) \tag{4}$$

In conclusion, one has:

$$P(T_i|\,E) = \frac{P(E|T_i)\,P(T_i)}{\sum_{j=1}^{n} P(E|T_j)\,P(T_j)} \tag{5}$$

Traced by a druidess?

Based on the above considerations, we devised an algorithm to calculate the probability that the person who traced the ellipse belongs to a given gender, employing Bayes' theorem (Stone 2013). The idea of carrying out such calculations was inspired by the consideration that, as stated above, while the height of 1.60 meters seemed compatible with a woman, 1.78 meters seemed compatible with a man.

Let us first calculate the conditional probability that a woman traced the site, if the site has the measured dimensions, P(tw|ms). For the calculation, it is necessary to know the conditional probabilities that the site was built with the given measurements, assuming that it was traced by a woman, P(ms|tw), or a man, P(ms|tm), and the a-priori probabilities that the site was traced by a woman (P_{tw}) or a man (P_{tm}).

One then has:

$$P(tw|ms) = (1 + \frac{P(ms|tm)}{P(ms|tw)}\frac{P_{tm}}{P_{tw}})^{-1} \quad (6)$$

To "let the facts speak for themselves", the a-priori probabilities P_{tw} and P_{tm} were both set equal to ½, with the result:

$$P(tw|ms) = (1 + \frac{P(ms|tm)}{P(ms|tw)})^{-1} \quad (7)$$

The probabilities P(ms|tw) and P(ms|tm) can then be calculated, provided that the height distribution of people in the 6^{th}-4^{th} centuries BC, the period to which we dated the site, is known. In (Koepke and Baten 2005) the distribution of heights for European women and men between the I and XVIII centuries AD was calculated. In the absence of more specific data, we assumed as average heights those relating to the I-V century AD, a reasonable assumption since, as Koepke and Baten report, the average height varied little until the threshold of the modern era, except for the Germanic contribution in the Early Middle Ages. The mean heights were calculated to be h_w=160±6 (women) and h_m=171±6 cm (men) and the height distribution is approximately Gaussian.

Regarding the calculation of P(ms|tw) and P(ms|tm), we observe that these terms contain two contributions, since the sub-optimal solutions we found make us think of steps used as units of measure, while the optimal solution suggests that a rope divided into 12 equal segments was employed, so as to form a right triangle with sides in the proportion 3:4:5. For a given gender "g", therefore, applying the definition of conditional probability one has:

$$P(ms|tg) = P(st \vee rp|tg) = P(st|tg) + P(rp|tg) \quad (8)$$

where the terms in the last equality are, respectively, the probability that the site was traced with the stride length of the druidess or druid as the unit of measure or by employing a rope divided into 12 equal segments.

Assuming that the structure was traced by a person of a given gender says nothing about the probability that a rope with knots was used, so we assume P(rp|tw)=P(rp|tm)=½. On the other hand, P(st|tw) and P(st|tm) are calculated by integrating over the height distributions of women and men, considering that the heights h_1=160 cm and h_2=178 cm, deduced from the unit of measurement used for dimensioning the site, are known with the precision of one centimeter and therefore determined within δ=±1 cm.

From Eq. (7) and (8) one then has:

$$P(tw|ms) = \left(1 + \frac{1+\frac{2\delta}{\sigma_m\sqrt{(2\pi)}}(\exp(\frac{-(h_1-h_m)^2}{2\sigma_m^2})+\exp(\frac{-(h_2-h_m)^2}{2\sigma_m^2}))}{1+\frac{2\delta}{\sigma_w\sqrt{(2\pi)}}(\exp[\frac{-(h_1-h_w)^2}{2\sigma_w^2}]+\exp(\frac{-(h_2-h_w)^2}{2\sigma_w^2}))}\right)^{-1} \quad (9)$$

In Eq. (9), σ_w and σ_m are the standard deviations of the height distribution of women and men, respectively, and it has been considered that δ is much smaller than h_1, h_2, h_w and h_m. Entering the necessary values in Eq. (9), the probability that a woman dimensioned the site turns out to be (55±8)%, where the error bar was obtained numerically, by varying h_1 and h_2.

Druidesses in Cisalpine Gaul? Part I

We also employed Bayes' theorem to estimate the probability that druids of both genders were present in Cisalpine Gaul. To this end, we first calculated the conditional probability that there were druidesses as well as druids in Cisalpine Gaul, assuming that the site was traced by a woman, P(wd|tw). For the calculation, we need to know the conditional probabilities that the site was traced by a druidess, assuming that in Cisalpine Gaul there were druidesses as well as druids, P(tw|wd), or only druids, P(tw|md), and the a-priori probabilities that in Cisalpine Gaul there were druids of both genders, (P_{wd}), or only male, (P_{md}). On this basis, one has:

$$P(wd|tw) = (1 + \frac{P(tw|md)}{P(tw|wd)} \frac{P_{md}}{P_{wd}})^{-1} \tag{10}$$

We then assume the following parameterization:

$$\frac{P(tw|md)}{P(tw|wd)} = \epsilon \tag{11}$$

where ε is infinitesimal, since if in Cisalpine Gaul there had been only male druids, the fact that the site was traced by a woman would be a mere “local fluctuation”.

We also assume:

$$P_{wd} = P(tw|ms) \tag{12}$$

$$P_{md} = 1 - P(tw|ms) \tag{13}$$

so that from Eq. (10) one gets:

$$P(wd|tw) = \frac{P(tw|ms)}{P(tw|ms)+\epsilon\,(1-P(tw|ms))} \tag{14}$$

As for the conditional probability that in Cisalpine Gaul there were druids of both genders, assuming that the site was traced by a man, P(wd|tm), we have:

$$P(wd|tm) = (1 + \frac{P(tm|md)}{P(tm|wd)} \frac{P_{md}}{P_{wd}})^{-1} \tag{15}$$

Assuming the parameterization:

$$\frac{P(tm|md)}{P(tm|wd)} = \mu \tag{16}$$

with the help of Eqs. (12) and (13) one has:

$$P(wd|tm) = \frac{P(tw|ms)}{P(tw|ms)+\mu\,(1-P(tw|ms))} \tag{17}$$

$$\mu = 2 \tag{18}$$

In Eq. (18), the factor 2 is due to the fact that P(tm|wd)=½, since the supposition that in Cisalpine Gaul there were druids of both genders does not provide us with any piece of information about the gender of the person who traced the structure and it is reasonable to assume that if only male druids existed in Cisalpine Gaul, then P(tm|md)=1.

We are now able to calculate the probability that there were druids of both genders in Cisalpine Gaul, P(wd), which has a different value as compared to the previously estimated P(wd), being an update of it. By the definition of conditional probability, in fact, one has:

$$\mathrm{P(wd) = P(wd|tw)P(tw) + P(wd|tm)P(tm)} \tag{19}$$

where for the a-priori probabilities we assume:

$$\mathrm{P_{tw} = \ P(tw|ms)} \tag{20}$$

$$\mathrm{P_{tm} = \ 1 - P(tw|ms)} \tag{21}$$

By combining Eqs. (14) and (17) through Eqs. (19), (20), (21), and since ε is infinitesimal and μ=2, one obtains:

$$\mathrm{P(wd) = P(tw|ms)\frac{3-2P(tw|ms)}{2-P(tw|ms)}} \tag{22}$$

Inserting the necessary values into the above equation, the probability that there were priestesses as well as priests in Cisalpine Gaul (whether druidesses and druids in the same sense as in other parts of the Celtic world, we cannot establish) is equal to (72±8) %. The error bar has been calculated numerically, by varying P(pw|ms).

Druidesses in Cisalpine Gaul? Part II

More significant than the calculation made in the previous section, in that it provides a "truth scale" based on psychometric studies (Good 1979), is the calculation of the Bayes factor (Kass and Raftery 1995) for druidesses and druids in Cisalpine Gaul. The Bayes factor for druidesses turns out to be:

$$K_w = \frac{P(tw|wd)}{P(tw|md)} = \frac{1}{\epsilon} \geq \sim 10 \tag{23}$$

while, for druids, it is:

$$K_m = \frac{P(tm|wd)}{P(tm|md)} = \frac{1}{\mu} = \frac{1}{2} \tag{24}$$

The meaning of the quantities introduced with Eqs. (23) and (24) is that, given the hypothesis that the structure was traced by a druidess (respectively, by a druid), the explanatory capacity of the interpretative models "druids and druidesses in Cisalpine Gaul" and "only druids in Cisalpine Gaul" is compared.

Starting from Eqs. (23) and (24), one then defines the following factor, a weighted average of the previously defined Bayes factors:

$$\mathrm{K = P(tw|ms)\ K_w + (1 - P(tw|ms))\ K_m} \tag{25}$$

The previous expression is obtained by considering that the fact that the structure was traced by a druidess or a druid leads us to favour the hypothesis that in Cisalpine Gaul there were "druids" of both genders for a factor equal to K_w or K_m, respectively; if we want to determine for which factor this hypothesis is globally favoured, we will have to calculate a weighted average of the two Bayes factors, with the weights given respectively by P(tw|ms) and P(tm|ms)=1-P(tw|ms). On this basis, one then obtains:

$$K = \frac{P(pw|ms)}{\epsilon} + \frac{(1 - P(pw|ms))}{\mu} \quad (26)$$

$$K \geq \sim \left(10\ P(pw|ms) + \frac{(1 - P(pw|ms))}{2}\right) \quad (27)$$

Substituting in Eq. (27) the necessary values results in a Bayes factor greater than ~(5.7±0.8), where the error bar was obtained numerically, by varying P(pw|ms). On the other hand, according to (Kass and Raftery 1995), we have the following interpretation for the previously calculated Bayes factor:

For 0<K<1 there is evidence to the contrary.

For 1<K<√10 there is marginal evidence.

For √10<K<10 there is substantial evidence.

For 10<K<100 there is strong evidence.

For K>100 there is decisive evidence.

Considering the previously calculated K-value and the fact that ε=10 is a conservative estimate for the value of the parameter, we can say that the measurements of the site, appropriately interpreted, lead to a "more than substantial" evidence for the hypothesis that both druids and druidesses (whether in the same sense as in Transalpine Gaul or the British Isles, we cannot determine) were present in Cisalpine Gaul.

Conclusions

Although our dating method and the consequent interpretation of the Barec as a high-altitude Celtic sanctuary were presented in (Gaspani and Spagocci 2018, 2020, 2022), we believe it is appropriate to report some considerations in support of it, since the hypothesis of the presence of druidesses and druids in Cisalpine Gaul is linked to this interpretation.

In the same Lombard valley (Val Brembana) of the Barec, a possible high mountain Celtic sanctuary was recognized by (Casini et al. 2010, p. 133-154) and dated to the VI century BC. Rock engravings were found in the same area, including many inscriptions (Fossati, Casini 2013, p. 377-392; Motta 2016). In protohistoric inscriptions, the Lepontic and Camunian languages are used, which seems to indicate that both Celts and Rhaetians converged in the area. These facts, in our opinion, reinforce the interpretation of the enclosure as an Iron Age Mountain sanctuary.

We also want to address the problem of the deviation from the elliptical shape of a part of the enclosure (Fig. 1). In this regard, we suggest that in Medieval times the structure, which had fallen into ruin, was reused as a fence for livestock; in fact, one of the authors (Gaspani 2001) reports that in the XIV century statutes of the Community of Averara, to which the Barec site belonged, the agricultural use of the enclosures in the area is mentioned. Having lost the sense of the sacredness of the place, the part that had fallen into disrepair may have been rebuilt without respecting its previous elliptical shape. In fact, visual examination has shown that the stones of the non-elliptical sector have a different appearance than the elliptical part.

Beyond what we believe to be the interest of the results obtained, which seem to be relevant not only for for Cultural Astronomy but also for the history of Cisalpine Gaul, we believe we have shown how the study of an archaeological site using astronomical methods, if conducted with adequate material and, above all, conceptual means, may reveal important and unexpected details about the history and structure of the site.

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References

Casini S., Longhi C., Castellano L., Croce E., Lando A. Un santuario celtico a Carona (Bergamo)? Ricerche e ritrovamenti nell'area del masso inciso CMS 1. Notizie Archeologiche Bergomensi, 2010, no. 18, pp. 133-154.

Cernuti S., Gaspani A. Introduzione all'Archeoastronomia: nuove tecniche di analisi dei dati, Firenze, Tassinari, 2006.

Gaspani A. I Barec del Monte Avaro: Semplici Recinti di Pastori? Terra Insubre, 2001, no. 18, pp. 14-17.

Gaspani A. Il calendario di Coligny. Misura del tempo presso i Celti, Keltia, 2012.

Gaspani A., Cernuti S. L'Astronomia dei Celti, Aosta, Keltia Editrice, 1997, 175 p.

Gaspani A., Spagocci S. Il Barec dei Piani del Monte Avaro. Terra Insubre, 2018, no. 85, pp. 36-43.

Gaspani A., Spagocci S. Archaeoastronomical "Stratigraphy": Investigations on a Cisalpine Celtic Enclosure. SEAC Publications, 2020, no. 1, pp. 126-134.

Gaspani A., Spagocci S. A Method for Dating Archaeological Structures Based on Astronomical Alignments. Archaeoastronomy and Ancient Technologies, 2022, vol. 10, no. 1, pp. 79-90.

Good I.J. Studies in the History of Probability and Statistics. XXXVII A.M. Turing's Statistical work in World War II. Biometrika, 1979, vol. 66, no. 2, pp. 393–396.

Fossati A. E., Casini S. Incisioni rupestri e iscrizioni preromane a Carona, Val Brembana (Bergamo). Bullétin d'Etudes Préhistoriques et Archéologiques Alpines, 2013, no. 24, pp. 377-392.

Kass R., Raftery A. Bayes Factors. Journal of the American Statistical Association, 1995, vol. 90, no. 430, pp. 773-795.

Koepke N., Joerg B. The Biological Standard of Living in Europe During the Last Two Millennia. European Review of Economic History, 2005, v. 9, no. 1, pp. 61-95.

Maddalena E. Orienteering. Elements of Orientation and Topography for excursions, mountaineering, hiking, survival, soft air and orientation race. Milano: Hoepli, 2010.

Motta F. Sulle Ultime Campagne di Studio delle Epigrafi Preromane di Carona. Gli Orobi. Alle radici delle genti bergamasche Varese: Terra Insubre, 2016.

Stone J. Bayes' rule: a tutorial introduction to Bayesian analysis, Sheffield, Sebtel Press, 2013.

Figure 1. A view of the Mt Avaro Barec (Gaspani and Spagocci 2020). (From Google Earth, image © 2020 Maxar Technologies).

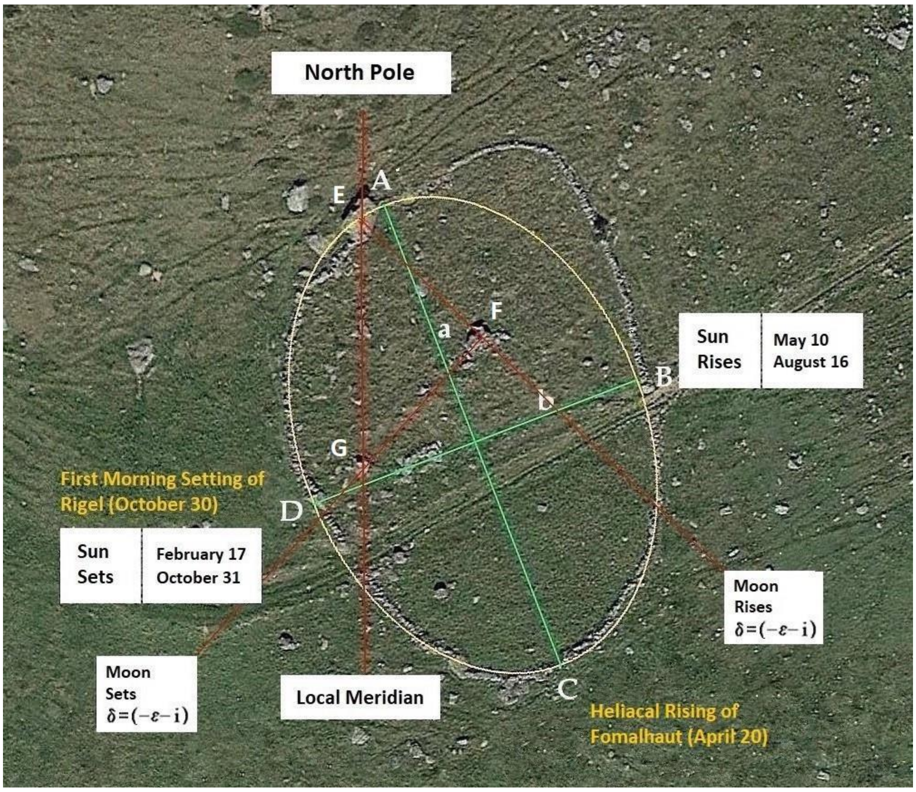

Figure 2. Alignments of the Mt Avaro Barec structures (Gaspani and Spagocci 2020). (From Google Earth, image © 2020 Maxar Technologies

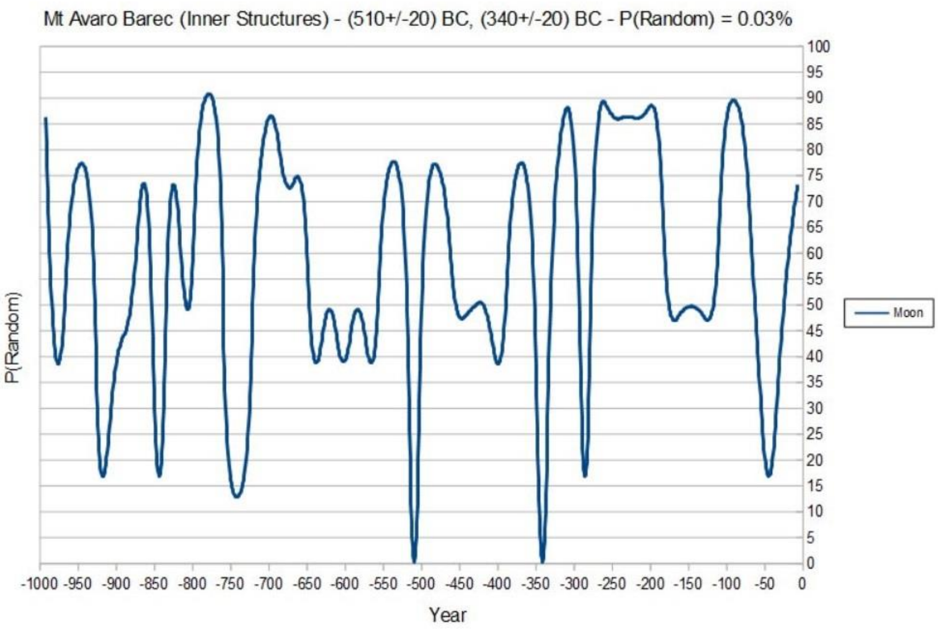

Figure 3. Dating curve, based on randomness probability, for the Barec's inner structures, according to their lunar alignments (Gaspani and Spagocci 2020).

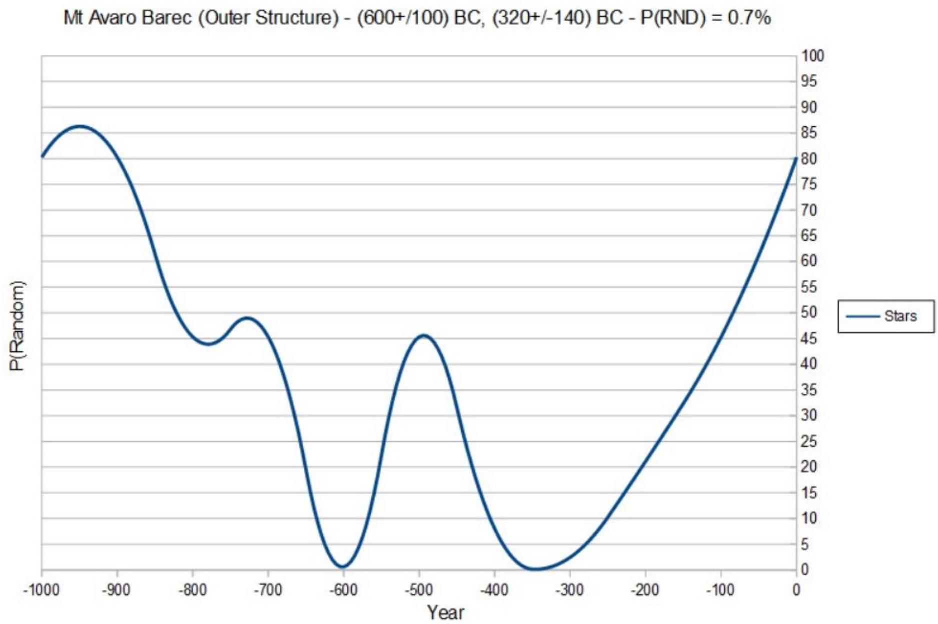

Figure 4. Dating curve, based on randomness probability, for the Barec's outer structures, according to their stellar alignments (Gaspani and Spagocci 2020).